IUTAM Symposium on 50 Years of Chaos : Applied and Theoretical
Kyoto, Japan, November 28 - December 2, 2011

Hierarchical structural analysis for the multi-folding structures with hill-top bifurcation points

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Abstract

This paper reviews the theoretical basis for the dynamic numerical analysis to examine the elastic stability of the multi-folding microstructure(MFM) [1]. The analysis allows for geometrical non-linearity with localization based upon bifurcation theory [2]. Comparisons are made between experimental folding patterns and the patterns obtained from the numerical method in which bifurcations are demonstrated as elastic unstable behaviour (e.g. [3]). We have obtained interesting bifurcation paths from the hill-top singular point on a diamond truss which is one of the folding multi-layered truss in a plane. The author suggest that the understanding of snapthrough behaviour for folding mechanics in 3D will be very useful for the development of lightweight structures subject to bifurcation of the hill-top branching type.

Nonlinear elastic folding analysis for a multi-layered truss

We consider the folding mechanism for the pantographic folding structures subject to the periodic loadings at the top of the multi-folding system. Let us consider the analytically ideal model of the multi-layered trusses based on the MFM. No allowance is made the friction, gravity or buckling for each member by itself.

Theoretical approach for the MFM model

We consider that a structural system contains several pin-joined diamond trusses with the left-right symmetry. In general, it is more complex to make the formula of the equilibrium equations with geometrical condition. It looks that solving the equations are more difficult. In particular, if we used the engineering strain for the total strain energy, it would be so far from theoretical work with mathematical proof. Hence, for example, even though the system has beautiful symmetry, it has never been so useful to know the reason why it happens on symmetry-breaking issue. In case of this model, we have to consider the theoretical bifurcation analysis including with nonlinear dynamics.

Now, let us consider a theoretical estimation for the multi-folding truss model. We assume a periodic height for each layer of $h_i = \gamma_i L$ where the width L of the truss is fixed. Therefore, an initial length for each bar in the geometry of the figure is expressed as $\ell_i = \sqrt{L^2 + h_i^2} = L\sqrt{1+\gamma_i^2}$, $(i = 1, \cdots, n)$. The deformed length of each bar denoted as $\hat{\ell}_i$, is a function of the height and the nodal displacement variables

$$\hat{\ell}_i = L\sqrt{1 + (\gamma_i - q_i(\bar{v}_i, \bar{v}_{i+1}))^2}\ , \quad i = 1, \cdots, n \tag{1}$$

where $\gamma_i = h_i/L > 0$, $\quad q_i(\bar{v}_i, \bar{v}_{i+1}) = \bar{v}_i - \bar{v}_{i+1}$, $q_n(\bar{v}_n, \bar{v}_{n+1}) = \bar{v}_n$, $\bar{v}_i = v_i/L$, $\quad (i = 1, \cdots, n)$.

Using Green's expression for strain we apply the strain in each elastic bar as

$$\varepsilon_i \equiv \frac{1}{2}\left\{\left(\frac{\hat{\ell}_i}{\ell_i}\right)^2 - 1\right\} = \frac{1}{2}\left\{\frac{1 + (\gamma_i - q_i)^2}{1+\gamma_i^2} - 1\right\}, \tag{2}$$

The total energy of a half of the system is given by

$$\mathcal{V} = \sum_{i=1}^{n} \frac{EA_i\ell_i}{2}(\varepsilon_i)^2 - f^*\bar{v}_1 L = \sum_{i=1}^{n} \frac{EA_i L\sqrt{1+\gamma_i^2}}{2}\frac{1}{4}\left\{\frac{1+(\gamma_i - q_i)^2}{1+\gamma_i^2} - 1\right\}^2 - \frac{f}{2}\bar{v}_1 L \tag{3}$$

in where, f^* is a half of load $f^* = f/2$, and $\bar{v}_{n+1} = 0$. In the case of the same height for each layer, $\gamma = \gamma_i$ and the same stiffness for each member, $EA = EA_i$. Then, the total potential energy can be described as

$$\mathcal{V} = \frac{\beta L}{8}\sum_{i=1}^{n}(q_i)^2(q_i - 2\gamma)^2 - \frac{f}{2}\bar{v}_1 L \tag{4}$$

where the stiffness parameter $\beta = EA/(1+\gamma^2)^{3/2}$, which is a function of γ. From Eq. (4), we can obtain the equilibrium equations based on the principal of minimum energy[2].

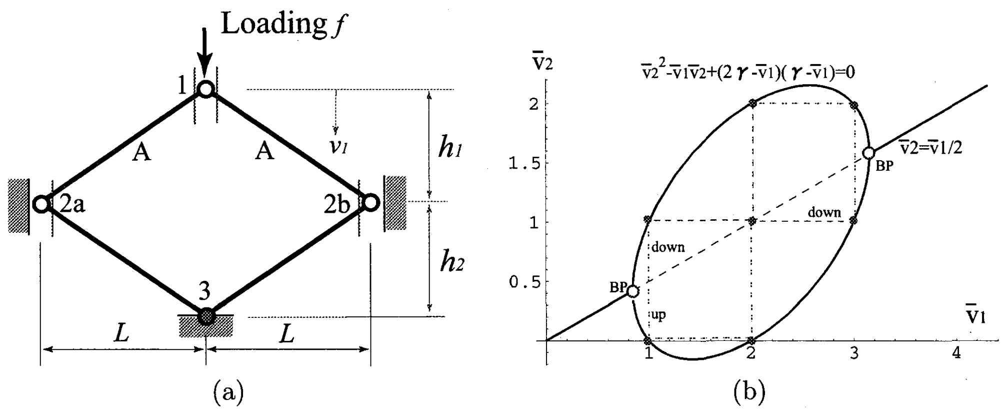

Figure 1: Mathematical model for a diamond truss ; (a) a model ($n = 2$), (b) Equilibrium paths for the model.

Nonlinear equilibrium equations

By allowing for symmetric model, we can therefore consider nonlinear equilibrium equations based on the total strain energy theoretically.

For the 1st, and $i-$th equilibrium equations chained

$$F_i \quad = \quad \frac{\partial \mathcal{V}}{\partial v_i} = \sum_{j=1}^{2} \frac{\partial \mathcal{V}}{\partial q_j} \frac{\partial q_j}{\partial \bar{v}_i} \frac{1}{L} = \beta \sum_{j=1}^{2} q_j (q_j - \gamma)(q_j - 2\gamma) = 0 \tag{5}$$

For example, let us consider $n = 2$ as a diamond truss. These equations equal 0 in the limit value problem of total potential energy. It is possible to solve all variables $\bar{v}_i$ by substituting the obtained solutions into the next limited condition using the theorem of implicit function.

$$F_2(\bar{v}_1, \bar{v}_2) = 0 \rightarrow \bar{v}_2 = \mathcal{F}_2(\bar{v}_1); \quad F_1(\bar{v}_1, \bar{v}_2) = F_1(\bar{v}_1, \mathcal{F}_2(\bar{v}_1)) = 0 \ , \tag{6}$$

where $\mathcal{F}(\cdot)$ denotes a function of the nonlinear solutions. Finally, we obtain all equilibrium solutions completely.

Formula for the Duffing dynamic system

It is well known that dynamic analysis and techniques with numerical work also, and for example, a simple truss [3, 4] has one of strange attractor models of nonlinear dynamics. The dynamic analysis equation for the folding truss combines mass, damping and nonlinear stiffness $\{F_i(\boldsymbol{v})\}^{\mathrm{T}} = \boldsymbol{F}(\boldsymbol{v}) \in \mathbf{R}^{\mathrm{N}}$ in the following equation:

$$M\ddot{\bar{\boldsymbol{v}}}(t) + C\dot{\bar{\boldsymbol{v}}}(t) + \boldsymbol{F}(\bar{\boldsymbol{v}}(t)) = 0,$$

where, $M \in \mathbf{R}^{N\times N}$ is the mass matrix; $C \in \mathbf{R}^{N\times N}$ is the damping; $\boldsymbol{F}(\cdot)$ is the nonlinear stiffness; $\{\ddot{\bar{v}}_i(t)\}^{\mathrm{T}} = \ddot{\bar{\boldsymbol{v}}}(\mathrm{t}) \in \mathbf{R}^{\mathrm{N}}$ is acceleration; $\{\dot{\bar{v}}_i(t)\}^{\mathrm{T}} = \dot{\bar{\boldsymbol{v}}}(\mathrm{t}) \in \mathbf{R}^{\mathrm{N}}$ is the velocity; $\{\bar{v}_i(t)\}^{\mathrm{T}} = \bar{\boldsymbol{v}}(\mathrm{t}) \in \mathbf{R}^{\mathrm{N}}$ is the displacement.

Remarks and Acknowledgement

This paper clearly shows that there are different bifurcation paths and a primary nonlinear equilibrium path with a theoretical solution approach. And there is a hill-top bifurcation point or there are multiple bifurcation equilibrium paths through two BPs. This nonlinear phenomena appears to be folding behaviour that limits the holding supports in a real structure like an umbrella. In future, we will show the numerical results as attractors for nonlinear strange behaviour.

This research of "multiple symmetry-breaking" has been getting *Grant-in-Aid for challenging Exploratory Research* of Japan Society for the Promotion of Science (JSPS). It is so grateful to support the scietific research.

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